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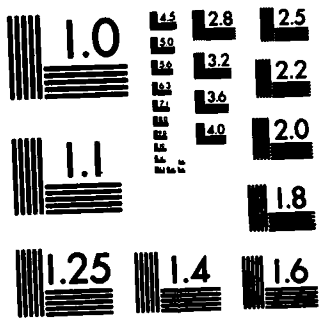

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19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

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20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

The main thrust of the research has been toward the development of efficient algorithms for solving the finite - dimensional constrained optimization problem

$$\text{minimize } f(x)$$
$$\text{subject to } g(x) \leq 0 \text{ and } h(x) = 0.$$

where $f:R^n \to R$, $g:R^n \to R^m$, and $h:R^n \to R^p$ are smooth functions. Historically, problems of this type have been solved by either penalty function methods or through linearization procedures. The fact that neither of these techniques is

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20. ABSTRACT CONTINUED

completely satisfactory for general nonlinear problems has lead to a concentrated research effort to find better approaches. What has so far emerged from this work is a blending of the penalty function and linearization ideas with the quadratic approximation methods associated with unconstrained optimization. While there remain many unresolved issues, it is now apparent that this synthesis has resulted in more efficient algorithms for the nonlinear constrained optimization problem.

FINAL REPORT

P.T. Boggs and J.W. Tolle

December 23, 1982

U.S. Army Research Office

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The University of North Carolina at Chapel Hill

This report contains a summary of the research on nonlinear optimization conducted at the University of North Carolina and the Army Research Office from April 15, 1977 through September 30, 1982. This research was sponsored under grants DAAG 29-77G0125, - 79G0014, - 79D1002 from the U.S. Army Research Office.

Introduction

The main thrust of the research conducted during this period has been toward the development of efficient algorithms for solving the finite - dimensional constrained optimization problem

$$\text{minimize} \quad f(x)$$
$$\text{subject to } g(x) \leq 0 \text{ and } h(x) = 0.$$

where $f: \mathbb{R}^n \to \mathbb{R}$, $g: \mathbb{R}^n \to \mathbb{R}^m$, and $h: \mathbb{R}^n \to \mathbb{R}^p$ are smooth functions.

Historically, problems of this type have been solved by either penalty function methods or through linearization procedures. The fact that neither of these techniques is completely satisfactory for general nonlinear problems has led to a concentrated research effort to find better approaches. What has so far emerged from this work is a blending of the penalty function and linearization ideas with the quadratic approximation methods associated with unconstrained optimization. While there remain many unresolved issues, it is now apparent that this synthesis has resulted in more efficient algorithms for the nonlinear constrained optimization problem. The research described briefly below has been part of this development.

Penalty functions

A family of multiplier functions, $M(x,\lambda;c,d)$, has been developed for the equality-constrained problem. M depends upon two parameters (c,d) and is concave and quadratic in the multiplier λ. For certain finite values of c and d the pair (x^*,λ^*) is a saddle point of M if and only if (x^*,λ^*) is an optimal vector and its multiplier for the problem. If $\nabla_\lambda M(x,\lambda;c,d) = 0$ is solved for λ in terms of x, then for appropriate values of c and d, $M(x,\lambda(x); c,d)$ is an exact penalty function for the optimization problem. The penalty function and its applications are discussed in [1T], [2T], [3T], [6T], [1P], and [2P].

Constrained optimization and differential equations.

It is well known that for the unconstrained minimization, solving the gradient differential equation, $x' = -\nabla f(x)$, $x(0) = x^\circ$, by Euler's method is equivalent to using a steepest descent algorithm to minimize f. In [5T], [7T] and [1P] an algorithm for solving the equality-constrained problem is developed which is based on the gradient system

$$x' = -\nabla_x M(x,\lambda;c,d) \ , \quad x(0) = x^\circ \ ,$$

$$\lambda' = -\nabla_\lambda M(x,\lambda;c,d) \ , \quad \lambda(0) = \lambda^\circ \ .$$

where M is the multiplier function reported above. Because of the large values of the parameter c which are required to validate the important theoretical properties of M, the above system is best analyzed as a singularly perturbed system of the form

$$\frac{1}{c} x' = \psi(x,\lambda;c,d)$$

$$\lambda' = \psi(x,\lambda;c,d) \ .$$

Quasi Newton methods.

An important new technique for solving the constrained nonlinear problem is the method of successive quadratic programming. Locally this method is equivalent to solving the nonlinear system

$$\nabla_x \ell(x,\lambda) = 0 \ ,$$

$$h(x) = 0 \ ,$$

$$g_i(x) = 0 \ , \ i \in I$$

where $\ell(x,\lambda)$ is the Lagrangian function and I is the index set of active inequality constraints at optimality. In a series of reports and papers, [4T], [8T], [3P], and [4P] the local convergence of quasi-Newton algorithms for solving this system is investigated. In particular, necessary conditions for convergence and necessary and sufficient conditions for superlinear convergence are obtained.

Implementation

Research continues on the implementation of an efficient algorithm which is both robust and locally superlinearly convergent. One major difficulty which has been overcome in this quest is the identification of a merit function (a function used to measure the progress of the algorithm) which is consistent with superlinear convergence. In [9T], [10T], [5P], and [6P] a member of the family of multiplier functions described above is shown to have this desirable property. This merit function is incorporated into an algorithmic scheme for solving the nonlinear problem.

Continuing investigations.

The research initiated under the sponsorship of the Army Research Office is continuing. Current topics of interest include the application of successive quadratic programming to large scale nonlinear problems [11T] and the implementation of the algorithm described in [6P] in a more efficient and robust manner [12T].

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